Numerical Method for Hypersonic Internal Flow over Blunt Leading Edges and Two Blunt Bodies

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The steady-state solution of mixed subsonic-supersonic flow near a blunt leading edge in hypersonic internal flow and supersonic flow over two blunt bodies is obtained as an asymptotic solution of the unsteady equations of motion by a time-dependent method. The continuous flow region between the shock and the body is calculated by the method of finite differences and exact boundary conditions are applied using the two-dimensional, unsteady, method of characteristics. Numerical results are presented for two-dimensional and axisymmetric flows with a continuously curved shock and shock interacting at the axis of symmetry as Mach or regular interaction. The shock wave configuration is found to be dependent on freestream Mach number, leading edge shape and bluntness. Theoretical calculations for inviscid, perfect gas are compared with experimental data, obtained in a hypersonic gun tunnel at a freestream Mach number of 8.33, for leading edge shapes of circular cross section.

Nomenclature

a = nondimensional sonic speed
A = quantities defined by Eqs. (5, 6, and 8)
b = radial distance of the body
B = quantities defined by Eqs. (5, 6, and 8)
C = quantity defined by Eq. (4a)
d = twice the leading edge nose radius
D = $2H$
f = dummy variables representing general fluid property
F = quantity defined by Eq. (4b)
H = distance of the origin of toroidal coordinates from the axis of symmetry
j = constant, which is zero for two-dimensional flow and unity for axisymmetric flow
M = Mach number
p = nondimensional static pressure
P = $\ln p$
r = radial toroidal coordinate
R = $\ln \rho$
s = radial distance of shock or axis of symmetry in physical coordinates
S = $P - \gamma R$ = nondimensional entropy
t, T = nondimensional time in physical and transformed coordinate systems, respectively
u, v = nondimensional velocity components in r and θ direction, respectively
U, V = nondimensional velocity components in ξ and η direction, respectively
W = shock wave velocity in r direction
x, y = abscissa and ordinate, respectively, in Cartesian frame in the physical plane
X, Y = abscissa and ordinate, respectively, in Cartesian frame in the transformed plane
γ = ratio of specific heats
δ = $s - b$
$\Delta T, \Delta X, \Delta Y$ = mesh spacings for time T and coordinates X and Y, respectively
$\Delta\lambda$ = physical distance between nodal points
$\Delta\varepsilon$ = standoff distance
η = coordinate, tangential to boundary
θ = polar angle in toroidal coordinates
ξ = coordinate, normal to boundary
ρ = nondimensional density
σ = quantity defined by Eqs. (5, 6, and 8)
ϕ = angle between r and ξ axis

Subscripts

0 = reference values
b = body
A, B, C = body points
c = values in Cartesian coordinate system in x, y plane
I = computational region I
L = computational region L
n = normal component
st = stagnation values
∞ = freestream values

Superscripts

m = position of nodal point in Y direction
n = position of nodal point in X direction
$*$ = values at the point in the flowfield at time T, which are used in the compatible equation (14)
$(\bar{\ })$ = physical variable

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I. Introduction

THE supersonic flow over a single blunt body has been solved by different methods. These include the inverse methods,[1,2] the method of integral relations,[3] the method of finite differences,[4,5] and the method of characteristics.[6–8] A review of the different methods is given by Hayes and Probstein[9] and Belotserkovskii and Chunshkin.[10] A time-dependent solution of the two-dimensional and axisymmetric blunt body flows is obtained by Moretti and Abbett[11] and three-dimensional flow by Moretti and Bleich.[12] The physical and numerical nature of the time-dependent method described in Ref. 11, is discussed by Anderson et al.[13]

The supersonic flow over two blunt bodies or at the entrance of an internal compression inlet can have three distinct configurations (Fig. 1). A) Continuously curved shock; this occurs with a strong shock, low freestream Mach numbers and highly constricted inlets. B) Shock with Mach interaction; this occurs with intermediate values of the aforementioned parameters. C) Shock with regular interaction; this occurs with a weak shock, high freestream Mach numbers and mildly constricted inlets. This configuration does not exist for axisymmetric flows.

For blunt leading edges, the flow is locally subsonic at the leading edge and becomes supersonic at the inner and outer sides. The governing equations of motion describing the steady flow are elliptic in the local subsonic region and hyperbolic in the adjacent supersonic regions. For mixed, initial value problems concerning steady flow, we can consider the unsteady equations of motion describing the transient flowfield, and in doing so the partial differential equations become hyperbolic in the entire flow region, thus permitting a time-dependent solution. The steady-state flow is then obtained as an asymptotic solution, with time, from assumed initial conditions. This paper concerns the development and use of a time-dependent method for the solution of hypersonic internal flow over a blunt leading edge and supersonic flow over two blunt bodies. Two-dimensional symmetric and axisymmetric flows are considered with configurations of continuously curved shock and shock interacting at the axis. The leading edge shape is prescribed and the shock wave is considered as a moving discontinuity. The continuous flow region between the shock and the body is treated by the method of finite differences and exact boundary conditions are applied using the method of characteristics. The solution is confined to the subsonic and adjacent supersonic flow regions near the blunt leading edge. The shock interaction at the axis is not included in the analysis. Numerical and experimental results are compared.

II. Equations of Motion in Physical and Transformed Coordinates

Basic Equations in Physical Coordinates

The basic conservation equations for two-dimensional and axisymmetric inviscid flow of perfect gas in toroidal coordinates (Fig. 2) are

$$\frac{\partial R}{\partial t}+\frac{\partial u}{\partial r}+u\frac{\partial R}{\partial r}+\frac{u}{r}+\frac{1}{r}\frac{\partial v}{\partial \theta}+\frac{v}{r}\frac{\partial R}{\partial \theta}+j\frac{(u\sin\theta+v\cos\theta)}{(H+r\sin\theta)}=0 \tag{1a}$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial r}+\frac{v}{r}\frac{\partial u}{\partial \theta}-\frac{v^2}{r}+\frac{p}{\rho}\frac{\partial P}{\partial r}=0 \tag{1b}$$

$$\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial r}+\frac{v}{r}\frac{\partial v}{\partial \theta}+\frac{uv}{r}+\frac{1}{r}\frac{p}{\rho}\frac{\partial P}{\partial \theta}=0 \tag{1c}$$

$$\partial S/\partial t+u(\partial S/\partial r)+(v/r)(\partial S/\partial \theta)=0 \tag{1d}$$

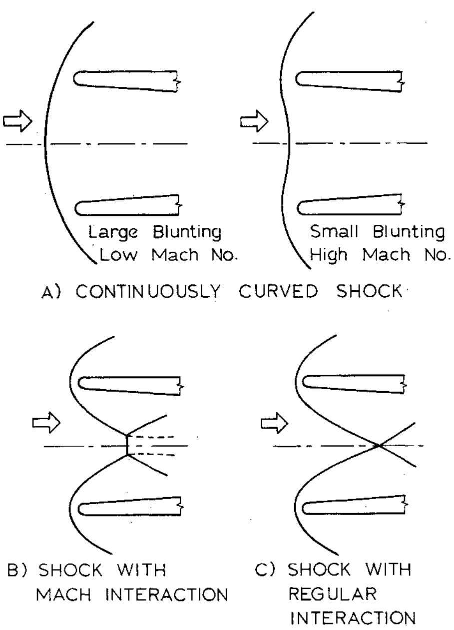

Fig. 1 Types of hypersonic internal flow with blunt leading edge.

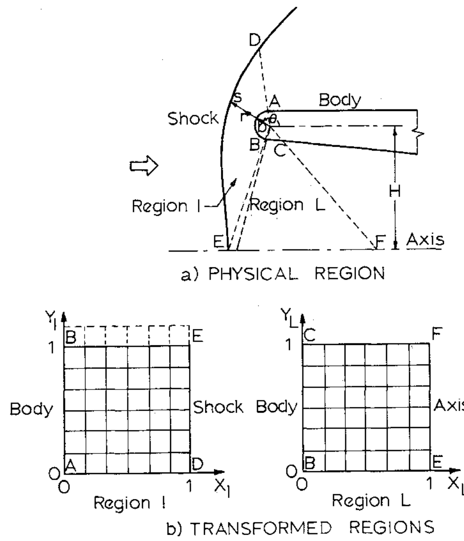

Fig. 2 Coordinate system for continuously curved shock.

The aforementioned equations contain all the variables in nondimensional form, where the distances are nondimensionalized in terms of a characteristic length $\bar{r}_0$, pressure by $\bar{p}_\infty$, density by $\bar{\rho}_\infty$, velocity by $(\bar{p}_\infty/\bar{\rho}_\infty)^{\frac{1}{2}}$ and time by $\bar{r}_0(\bar{\rho}_\infty/\bar{p}_\infty)^{\frac{1}{2}}$.

Transformation for Continuously Curved Shock (Case A)

In order to evaluate the partial derivatives in the continuous flow region, the physical region is transformed into a rectangular computational region, with a suitable transformation of the independent variables. The boundaries of the physical region form the sides of the rectangle, and the time-dependent physical boundaries become fixed in the computational region. For continuously curved shock (Fig. 2a), the flow region is bounded by the shock DE, the body AC, the axis of symmetry EF, and the chosen supersonic flow boundaries AD and CF. It is not possible to transform this physical region into a single rectangular computational region. Instead, it is transformed into two rectangular regions, with an interface BE which moves in the physical region but remains fixed with time in the computational region. The transformation for the independent variables for region I, containing the shock and body, and for region L, containing the axis of symmetry and body (Fig. 2b) are

$$X_I=\frac{r-b(\theta)}{s(\theta,t)-b(\theta)},\quad Y_I=\frac{\theta-\theta_A}{\theta_B(t)-\theta_A},\quad T=t \tag{2a}$$

$$X_L=\frac{r-b(\theta)}{s(\theta)-b(\theta)},\quad Y_L=\frac{\theta-\theta_B(t)}{\theta_C-\theta_B(t)},\quad T=t \tag{2b}$$

The equations of motion [Eq. (1)], in terms of these new independent variables, in the transformed region are

$$\frac{\partial R}{\partial T}+B\frac{\partial R}{\partial X}+A\frac{\partial R}{\partial Y}+\frac{1}{\delta}\frac{\partial u}{\partial X}+\frac{C}{r\delta}\frac{\partial v}{\partial X}+\frac{1}{r\sigma}\frac{\partial v}{\partial Y}+\frac{u}{r}+jF=0 \tag{3a}$$

$$\frac{\partial u}{\partial T}+B\frac{\partial u}{\partial X}+A\frac{\partial u}{\partial Y}-\frac{v^2}{r}+\frac{1}{\delta}\frac{p}{\rho}\frac{\partial P}{\partial X}=0 \tag{3b}$$

$$\frac{\partial v}{\partial T}+B\frac{\partial v}{\partial X}+A\frac{\partial v}{\partial Y}+\frac{uv}{r}+\frac{C}{r\delta}\frac{p}{\rho}\frac{\partial P}{\partial X}+\frac{1}{r\sigma}\frac{p}{\rho}\frac{\partial P}{\partial Y}=0 \tag{3c}$$

$$\partial S/\partial T+B(\partial S/\partial X)+A(\partial S/\partial Y)=0 \tag{3d}$$

where

$$C=(X-1)db/d\theta-X(\partial S/\partial\theta) \tag{4a}$$

$$F=(u\sin\theta+v\cos\theta)/(H+r\sin\theta) \tag{4b}$$

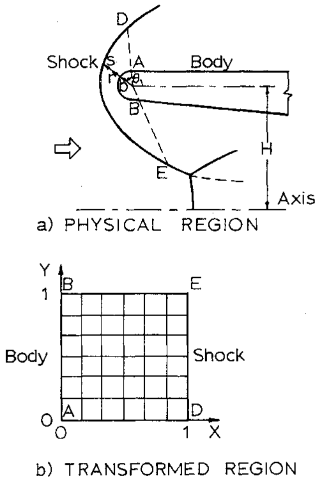

Fig. 3 Coordinate system for shock with regular or Mach interaction.

For region I,

$$X = X_I, \qquad Y = Y_I, \qquad \sigma = \theta_B - \theta_A$$
$$A = (1/\sigma)[v/r - Y(d\theta_B/dT)]$$
$$B = (1/\delta)(-WX + u + vC/r) \tag{5}$$

For region L,

$$X = X_L, \qquad Y = Y_L, \qquad \sigma = \theta_C - \theta_B$$
$$A = (1/\sigma)\{v/r - [(\theta_C - \theta)/\sigma](d\theta_B/dT)\}$$
$$B = (1/\delta)(u + vC/r) \tag{6}$$

Transformation for Shock with Mach or Regular Interaction (Cases B and C)

For shock with Mach or regular interaction (Fig. 3), the physical region of flow near the leading edge can be transformed into a single rectangular computational region with the following transformation of independent variables.

$$X = [r - b(\theta)]/[s(\theta, t) - b(\theta)], \quad Y = (\theta - \theta_A)/(\theta_B - \theta_A) \qquad T = t \tag{7}$$

The equations of motion in terms of these new independent variables are given by Eq. (3), where,

$$\sigma = \theta_B - \theta_A, \qquad A = (1/\sigma)(v/r)$$
$$B = (1/\delta)(-WX + u + vC/r) \tag{8}$$

The j term in the continuity equation [Eq. (3a)] for the axisymmetric case is indeterminate at the axis. The indeterminate form can be evaluated with the aid of L'Hospital's rule:

$$F = (1/r)[(\partial u/\partial\theta)\sin\theta + (\partial v/\partial\theta)\cos\theta + u\cos\theta - v\sin\theta]\cos\theta + [(\partial u/\partial r)\sin\theta + (\partial v/\partial r)\cos\theta]\sin\theta \tag{9}$$

III. Description of the Time-Dependent Method

The computational region is subdivided into an orthogonal mesh with nodal points equally spaced in each direction. All nodes on the boundary are called boundary points and those not on the boundary are called interior points. The information at each nodal point is stored in the computer memory and is used for subsequent calculation. The numerical computation is carried out with step increments in time, starting from assumed initial conditions.

Interior Points

The method of finite differences is used to compute the flow properties at the interior points. Starting from known initial conditions at time T, any flow variable $f(r, \theta, t)$ at a subsequent time $T + \Delta T$ is obtained from the second-order Taylor expansion,[14]

$$f(T + \Delta T) = f(T) + [\partial f(T)/\partial T]\Delta T + [\partial^2 f(T)/\partial T^2][(\Delta T)^2/2] \tag{10}$$

The first derivatives of the dependent variable f, with respect to time T, are obtained directly from Eq. (3). The second derivatives are obtained by differentiating these equations with respect to X, Y, and T and then rearranging the terms.

The space derivatives are obtained by a centered difference scheme.[11] The time derivative of the shock velocity, W, is given by

$$\partial W/\partial T = [W(T + \Delta T) - W(T)]/\Delta T \tag{11}$$

Boundary Points

The importance of the exact treatment of boundary conditions in the numerical solution of hyperbolic equations is discussed by Moretti.[15] The exact boundary conditions are applied at the boundary points on shock, body, and axis of symmetry. The Rankine-Hugoniot relations are applied at the shock points and the conditions of normal velocity component relative to the surface being zero is applied at the body and axis of symmetry points. The conditions at the boundary points at time $T + \Delta T$ should be compatible with the flowfield at the previous time T, as determined by the unsteady equations of motion. This condition is applied by using the method of characteristics.

The equations of motion involve two space variables and one time variable and hence the method of characteristics in three independent variables has to be considered. Various schemes are available in the numerical solution, the major difference in these schemes is in the handling of the partial derivatives that exist in the characteristic relations. The scheme discussed by Moretti[16,17] and used in the solution of external flow over blunt bodies,[11,12] is adopted for the present problem.

A new orthogonal coordinate system (ξ, η, T) is defined at each boundary point with the η axis being tangential to the boundary (Fig. 4). The equations of motion for an inviscid, perfect gas can be written with nondimensional variables as,

$$\frac{\partial P}{\partial T} + U\frac{\partial P}{\partial \xi} + \gamma\frac{\partial U}{\partial \xi} = -V\frac{\partial P}{\partial \eta} - \gamma\frac{\partial V}{\partial \eta} - j\frac{\gamma v_c}{y} \tag{12a}$$

$$\partial U/\partial T + U(\partial U/\partial \xi) + (a^2/\gamma)(\partial P/\partial \xi) = -V(\partial U/\partial \eta) \tag{12b}$$

$$\partial V/\partial T + U(\partial V/\partial \xi) = -V(\partial V/\partial \eta) - (a^2/\gamma)(\partial P/\partial \eta) \tag{12c}$$

The j term for the axisymmetric case in Eq. (12a) is retained in terms of the Cartesian coordinate system (x, y, t), with the x-axis coinciding with the axis of symmetry.

Equation (12) describes the unsteady flow near the boundary points and the method of characteristics is used in their solution. In order to simplify the analysis, terms on the right-hand side of Eq. (12), containing the η derivatives, are considered as forcing terms[16,17] and are replaced by finite difference relations, which enables one to obtain the characteristic relations in the (ξ, T)

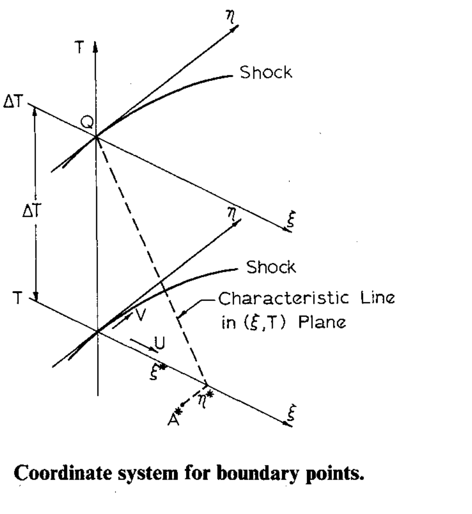

Fig. 4 Coordinate system for boundary points.

reference plane. The equations of the characteristic lines in the (ξ, T) plane are

$$d\xi/dT = U \pm a \tag{13}$$

and along these characteristic lines, the compatibility relations are

$$\frac{dP}{dT} \pm \frac{\gamma}{a}\frac{dU}{dT} = -\left[\gamma\frac{\partial V}{\partial \eta} + V\frac{\partial P}{\partial \eta} + j\frac{\gamma v_c}{y} \pm \frac{\gamma}{a}V\frac{\partial U}{\partial \eta}\right] \tag{14}$$

Shock Points

The shock wave is considered as a moving discontinuity. Its location at time $T + \Delta T$ is obtained from the shock velocity W at time T. The conditions behind the shock are obtained from the Rankine-Hugoniot relations for moving shock waves given by,

$$U = U_n\{[2^{\alpha} + (^{\alpha} - 1)U_n^2]/[(^{\alpha} + 1)U_n^2]\} - W\cos\phi \tag{15a}$$

$$\rho = U_n/(U + W\cos\phi) \tag{15b}$$

$$p = [(\gamma + 1)\rho - (\gamma - 1)]/[(\gamma + 1) - (\gamma - 1)\rho] \tag{15c}$$

$$V = U_\infty \sin(\theta + \phi) \tag{15d}$$

where

$$U_n = -U_\infty\cos(\theta + \phi) + W\cos\phi$$

$$\phi = \tan^{-1}[-(1/s)(\partial s/\partial\theta)] \tag{15e}$$

The new shock velocity at each shock point, at time $T + \Delta T$, is calculated by matching the conditions as obtained from the Rankine-Hugoniot relations at time $T + \Delta T$, with the flowfield at time T, using the method of characteristics. From the shock point a characteristic line, given by Eq. (13), is drawn in the (ξ, T) plane and the location of the point A^* (Fig. 4) at time T in the flowfield is obtained from the relations,

$$\xi^* = |U - a|\Delta T, \qquad \eta^* = -V\Delta T \tag{16a}$$

The conditions at A^* are obtained by interpolation from the known values at the adjacent nodes. These values are substituted in the compatibility relation, Eq. (14),

$$dU = \frac{a}{\gamma}dP + \left[a\left(\frac{\partial V}{\partial \eta} + \frac{jv_c}{y}\right) + V\left(\frac{a}{\gamma}\frac{\partial P}{\partial \eta} - \frac{\partial U}{\partial \eta}\right)\right]dT \tag{16b}$$

to obtain the velocity U at the shock point. If this value of the velocity component differs from that obtained by the Rankine-Hugoniot relations, the shock velocity W is changed accordingly and the process is iterated until convergence on U is reached.

Body Points

At the body points, the velocity component normal to the surface is zero. The method of calculation is similar to that described for the boundary points. From an assumed condition at the body point at time $T + \Delta T$, the location of the point A^* in time plane T is obtained from Eq. (13),

$$\xi^* = -(U + a)\Delta T, \qquad \eta^* = -V\Delta T \tag{17a}$$

The conditions at A^* are interpolated from known values at the adjacent nodes. These values are substituted in the compatibility relation, Eq. (14),

$$dP = -\frac{\gamma}{a}dU - \frac{\gamma}{a}\left[a\left(\frac{\partial V}{\partial \eta} + \frac{jv_c}{y}\right) + V\left(\frac{a}{\gamma}\frac{\partial P}{\partial \eta} + \frac{\partial U}{\partial \eta}\right)\right]dT \tag{17b}$$

to obtain the pressure P at the body. The particle isentropic condition of the fluid particle at time T, which would reach the body point at time $T + \Delta T$, is used to find the sonic speed, a, at the body point. The tangential velocity V at the body is obtained from the tangential momentum relation [Eq. (12c)]. The process is iterated until convergence on the location of point A^* is reached.

The axis of symmetry, in the case of the continuously curved shock, is treated as a particle path line and the analysis of the points on the axis is the same as for the body points, except that the proper signs in the characteristic equation and compatibility relation are taken into account.

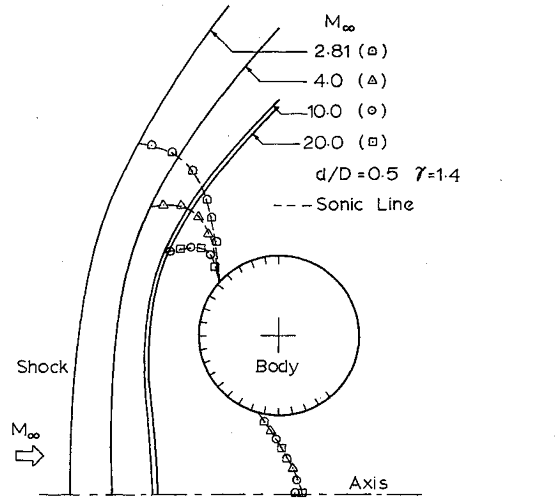

Fig. 5 Effect of freestream Mach number on shock shape; two circular cylinders.

Free Boundaries

No conditions at the chosen free boundaries, AD and CF, are known at the start of the solution. These boundaries are chosen far enough downstream to be in completely supersonic flow, so that the disturbances created at these boundary points are not propagated upstream. Hence the conditions at the free boundary points are obtained by extrapolation from the known data at the interior points.

Interface Points

The flow at the interface BE between the two computational regions, in the case of the continuously curved shock, is likely to be subsonic and hence these points cannot be treated in the same way as the free boundary points. The two regions are solved simultaneously, with a feedback from one region to the other, by having an additional row of points in the computational mesh (Fig. 2), the conditions at which are obtained from the known values in the adjacent region. The flow properties at the interface points are calculated by the method of finite differences, in the same way as the interior points.

Initial Conditions

The initial conditions are prescribed somewhat arbitrarily by assuming a shock shape and its location and initial distribution

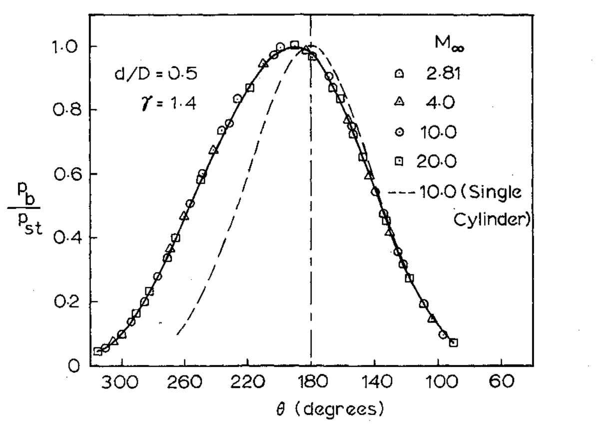

Fig. 6 Effect of freestream Mach number on static pressure distribution on the body; two circular cylinders.

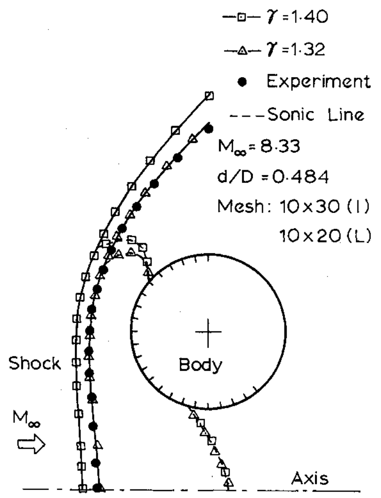

Fig. 7 Effect of specific heat ratio on shock shape; two circular cylinders with continuously curved shock.

of flow parameters in the computational region. The shape of the continuously curved shock is assumed to be normal between the axis of symmetry and the centerline of the leading edge, followed by a hyperbola outside of this region. The standoff distance between the shock and the leading edge is obtained from an empirical relation based on the results of the external flow over single blunt body.[2] The conditions at the shock points are calculated from the Rankine-Hugoniot relations. The stagnation streamline is assumed to be at the centerline of the leading edge with sonic points on the body at an angle of $\pi/4$ on the outer side and $\pi/2$ on the inner side of the centerline. A uniformly varying Mach number distribution is prescribed on the body with supersonic flow conditions at free boundaries. The axis of symmetry is treated as a streamline. Linear variation is assumed along the radial lines between the body and outer boundary in order to obtain the initial distribution of flow properties at the interior points.

For shock with Mach or regular interaction, the initial shock shape is assumed to be a hyperbola with conditions on either side of the leading edge centerline being symmetric.

Stability Criteria

The stability criteria for various difference schemes used in the numerical solution of hyperbolic differential equations are discussed by Heie and Leigh.[18] In the present problem a qualitative measure of stability of the difference method is based on the Courant-Friedrichs-Lewy (CFL) condition.[19] In order to satisfy the CFL stability condition for the interior and boundary points, a time step size T is chosen such that,

$$T = \text{minimum}|\Delta\lambda/[1.5a(M+1)]|$$

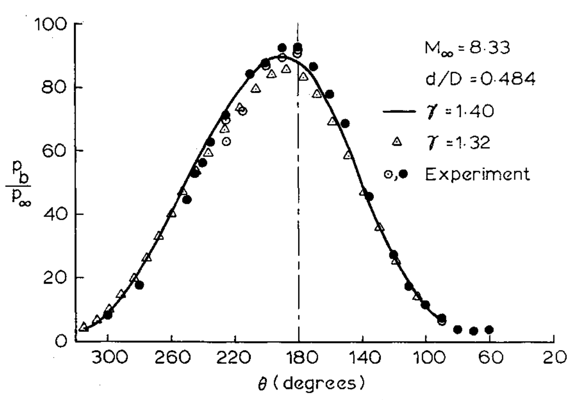

Fig. 8 Effect of specific heat ratio on static pressure distribution on the body; two circular cylinders with continuously curved shock.

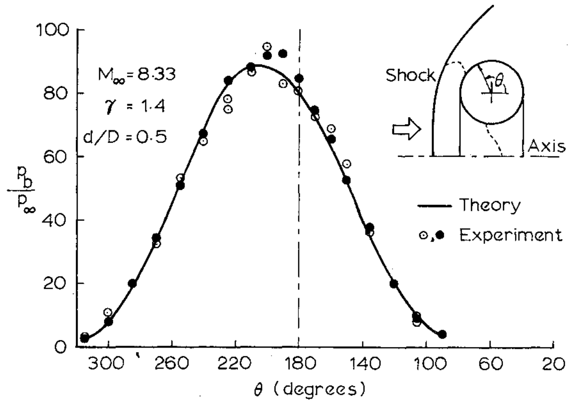

Fig. 9 Shock shape and static pressure distribution on the body (toroid); axisymmetric flow with continuously curved shock.

IV. Analysis of Results

Two digital computer programs are written to calculate the flow for Case *A* and the leading edge region of Cases *B* and *C*[20]. Although the programs are capable of handling any smooth leading edge shape, results are presented for only circular shapes. The solution is found to converge with time from assumed initial conditions. The convergence and stability of the numerical calculations are tested by using different mesh sizes for the same leading edge shape and freestream conditions. It is found that the steady state solution is not affected by a finer mesh size. The two-dimensional internal flow solution is identical to supersonic flow over two, two-dimensional blunt bodies influencing one another.

The effect of freestream Mach number for the same leading edge geometry is shown in Figs. 5 and 6. The standoff distance decreases with increase in Mach number. The corresponding shock shape changes from one with a continuous change in curvature having a maximum standoff distance at the axis to one with a dip at the axis having a maximum standoff distance at a distance away from the axis of symmetry. The surface pressure distribution is plotted as a ratio of static pressure to numerically observed stagnation pressure. It is interesting to note that the surface pressure distribution as well as the position of the sonic line at the body do not change appreciably with a change in freestream Mach number. The surface pressure distributions are compared with the theoretical values obtained for a single cylinder, from which the asymmetry in the case of two blunt body flow can be observed. The effect of specific heat ratio γ is shown in Figs. 7 and 8. There is a considerable change in shock position with a small variation in γ but the surface pressure distribution is not altered significantly.

The numerical results are compared with experimental data, obtained in the National Research Council of Canada Hypersonic Gun Tunnel, at a freestream Mach number of 8.33. The nominal

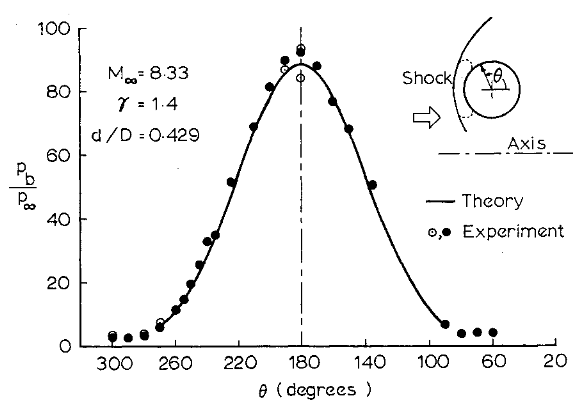

Fig. 10 Shock shape and static pressure distribution on the body (two circular cylinders); two-dimensional flow with shock interacting at the axis.

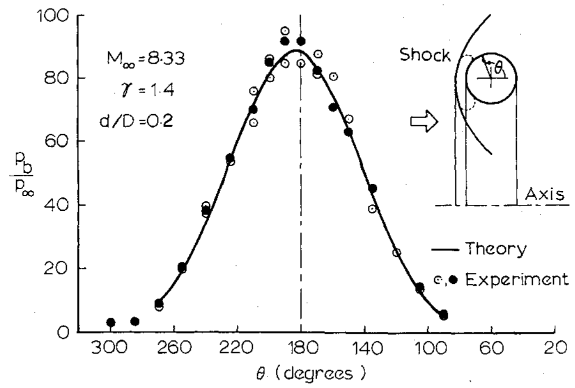

Fig. 11 Shock shape and static pressure distribution on the body (toroid); axisymmetric flow with shock interacting at the axis.

test section conditions used were, total temperature of 2430°R and total pressure of 4800 psi. Two-dimensional[21] and axisymmetric leading edges of circular cross section were tested.[20] The shock shape and surface pressure distribution for two-dimensional flow with continuously curved shock are shown in Figs. 7 and 8, respectively. The value of $\gamma = 1.32$ corresponds to the ratio of specific heats at the stagnation point temperature in the experimental set up. The shock wave is found to have a dip at the axis. The surface pressure distribution is asymmetric with respect to the leading edge of the cylinder and the stagnation point is 10° inboard toward the other cylinder. This is positive indication of interaction between the two cylinders. The stagnation pressure is slightly higher than the theoretical normal shock stagnation pressure and the streamline that wets the body does not pass through the normal portion of the bow shock. Results for axisymmetric flow are shown in Fig. 9. The results for shock with Mach interaction in the case of two-dimensional and axisymmetric flows are shown in Figs. 10 and 11, respectively. Filled and open symbols are used to depict the multiple measurements made at the same location on the body.

Only in the case of two-dimensional flow, for shock with Mach or regular interaction, the stagnation point is at the line of symmetry of the leading edge, with a symmetric pressure distribution on the inner and outer sides. For all other cases the stagnation point is at the inner side of the leading edge and the surface pressure distribution is not symmetrical. In all surface pressure measurements there is experimental scatter near the stagnation point of the leading edge causing a 4–5% discrepancy between theory and some measured points. Away from the stagnation point the scatter decreases, yielding a 1% difference (in relation to stagnation pressure) between theory and experiment.

V. Conclusions

The mixed subsonic-supersonic flow near a blunt leading edge, in hypersonic internal flow, is solved by a time-dependent method. It is a direct method where the body geometry and freestream conditions are specified. For the case of a continuously curved shock, the transformation of the physical region involves two rectangles, having a subsonic interface, which are solved simultaneously. The solution is found to converge asymptotically with time, from assumed initial conditions which do not influence the steady-state condition. The convergence and stability of the computation is not affected by a finer mesh size.

For continuously curved shock, in two-dimensional and axisymmetric flows, two types of shock shapes are obtained, depending on the freestream Mach number and bluntness; one with a continuous change in curvature having a maximum standoff distance at the axis of symmetry, the other with a dip at the axis having a maximum standoff distance at a distance away from the axis of symmetry. The latter shock wave is close to the transition from continuously curved shock to shock with Mach or regular interaction.

The stagnation point on the body is found at the inner side of the leading edge and the streamline that wets the body does not pass through the normal portion of the bow shock. The surface static pressure distribution is not symmetrical on the inner and outer sides except for two-dimensional flow with shock interacting at the axis, where the stagnation point is at the line of symmetry of the leading edge.

Theoretical calculations agree well with the experimental data for a freestream Mach number of 8.33.

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